Exam. Code : 103206
Subject Code : 1233

B.A./B.Sc. 6th Semester

MATHEMATICS

Paper—II

(Numerical Analysis)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Do any **FIVE** questions, selecting at least **TWO** questions from each section. All questions carry equal marks. Non-programmable scientific calculator is allowed.

SECTION—A

1. (a) Apply Bisection method in four stages to find the root of the equation $x^3 - 4x - 9 = 0$.

   (b) Show that Newton's method is of quadratic convergence. Find the cube root of 24 by applying Newton-Raphson formula.

2. (a) Find the real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places.

(b) Solve the system of equations $10x - 7y + 3z + 5u = 6$; $-6x + 8y - z - 4u = 5$; $3x + y + 4z + 11u = 2$; $5x - 9y - 2z + 4u = 7$ by Gauss elimination method.

3. (a) Solve by Jacobi's iteration method, the equations $20x + 4y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$.

(b) Using Gauss-Seidal iteration method solve the system of equations given in question 3(a).

4. (a) Prove that :

$$\nabla^2 y_8 = y_8 - 2y_7 + y_6;\ \delta^2 y_5 = y_6 - 2y_5 + y_4.$$

(b) Find the missing values in the following table :

x	45	50	55	60	65
y	3.0	–	2.0	–	– 2.4

5. (a) Prove with the usual notations that :

(i) $(E^{\frac{1}{2}} + E^{\frac{1}{2}})(1+\Delta)^{\frac{1}{2}} = 2 + \Delta$

(ii) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$

(b) Prove that :

$$u_1 x + u_2 x^2 + u_3 x^3 + \ldots\ldots = \frac{x}{1-x}u_1 + \left(\frac{x}{1-x}\right)^2 \Delta u_1 + \left(\frac{x}{1-x}\right)^3 \Delta^2 u_1 + \ldots\ldots\ldots$$

SECTION—B

6. (a) The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface. Find the value of y when x = 218 ft.

x = height	100	150	200	250	300	350	400
y = distance	10.63	13.03	15.04	16.81	18.42	19.90	29.27

(b) Given the values :

x	5	7	11	13	17
y	150	392	1452	2366	5202

evaluate f(x), using Lagrange's interpolation formula.

7. (a) Given that :

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}+\frac{d^2y}{dx^2}$ at $x=1.6$.

(b) Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2}\,dx$ by taking seven ordinates.

8. (a) Employ Taylor's method to obtain approximate values of y at x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x; y(0) = 0$. Compare the numerical solution obtained with the exact solution.

(b) Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2, 0.4.

9. (a) Interpolate by means of Gauss's backward formula, the population of a town for the year 1974, given that :

Year	1939	1949	1959	1969	1979	1989
Population (in thousands)	12	15	20	27	39	52

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using :

(i) Trapezoidal rule

(ii) Simpson's 3/8 rule.

10. (a) Using Picard's method, obtain a solution upto the fifth approximation of the equation $\frac{dy}{dx} = y + x$, such that y = 1, when x = 0. Check your answer by finding the exact particular solution.

(b) Apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.